Designing Resilient Electrical Distribution Grids

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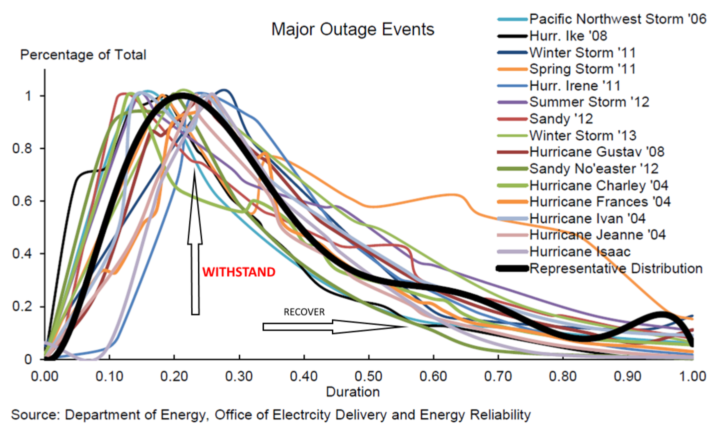
Presidential Policy Directive - Critical Infrastructure Security and Resilience

"The ability to prepare for and adapt to changing conditions and **withstand** and **recover** rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents."



A Simplified Model

Given a graph $G = (V, E)$ where V corresponds to node based upgrades (i.e. building facilities and microgrid generation capacity) and E corresponds to line based upgrades (i.e. building new lines, hardening lines and building switches) we want to find:

$$
\begin{aligned}
\min \quad & Budget(G') \\
\text{s.t.} \quad & G' \subseteq G \\
& T_s \subseteq G' && \forall s \in S \\
& T_s \in Trees(G) && \forall s \in S \\
& CriticalDemand(T_s) \geq MinCriticalDemand && \forall s \in S \\
& TotalDemand(T_s) \geq MinTotalDemand && \forall s \in S
\end{aligned}
$$

Algorithm 1: Greedy[1]

input: A set of disasters S;

for $s \in S$ **do**
- $\sigma^s \leftarrow Solve(P'(s))$;

$\sigma^*(x) = \max\{\sigma^s(x) | \forall s \in S\}$, $\forall x \in \mathcal{X}$;

Update $\sigma^*(x_i)$ with switches to preserve feasibility;

return σ^*

Algorithm 2: Scenario Based Decomposition[1]

input: A set of disasters S and let $S' = S_0$;

while $S \setminus S' \neq \emptyset$ **do**
- $\sigma^* \leftarrow$ Solve $P(S')$ exactly (SBD) or with VNS (SBVNDS);
- $I \leftarrow \langle s_1, s_2 \dots s_{|S \setminus S'|} \rangle s \in S \setminus S' : l(P'(s_i, \sigma^*)) \geq l(P'(s_{i+1}, \sigma^*))$;
- **if** $l(P'(I(0), \sigma^*)) \leq 0$ **then**
  - **return** σ^*;
- **else**
  - $S' \leftarrow S' \cup I(0)$;

return σ^*

Algorithm 3: Variable Neighborhood Search[1]

input: σ', MAXTIME, MAXRESTARTS and MAXITERATIONS;

Let $\sigma^{LP} \leftarrow Solve(P^{LP})$, $\sigma^* \leftarrow \sigma'$, $restart \leftarrow false$;

while $t <$ MAXTIME *and* $i <$ MAXRESTARTS **do**
- $j \leftarrow 0$;
- $n \leftarrow |x \in \mathcal{X} : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$;
- $J \leftarrow \langle \pi_1, \pi_2 \dots \pi_{|J|} \rangle \in \mathcal{X}$: $|\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$;
- **if** *restart* **then**
  - $i \leftarrow i + 1$;
  - $step \leftarrow \frac{4n}{d}$, $k = |\mathcal{X}| - step$;
  - *shuffle*(J)
- **else**
  - $step \leftarrow \frac{n}{d}$, $k = |\mathcal{X}| - step$;
- **while** $t <$ MAXTIME *and* $j \leq$ MAXITERATIONS **do**
  - $\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots, k)))$;
  - **if** $f(\sigma') < f(\sigma^*)$ **then**
    - $\sigma^* \leftarrow \sigma'$;
    - $i \leftarrow 0$;
    - $restart \leftarrow false$;
    - $j \leftarrow$ MAXITERATIONS;
  - **else**
    - $j \leftarrow j + 1$;
    - $k = k - \frac{step}{2}$;
    - **if** $j >$ MAXITERATIONS **then**
      - $restart \leftarrow true$;

return σ^*

[1]Yamangil, Bent, Backhaus (2015), Resilient Upgrade of Electrical Distribution Grids, *in proceedings of AAAI-15*, AAAI press.

Withstand

Develop new tools, methodologies, and algorithms to enable the design of resilient power distribution systems, using:

- asset hardening
- system design
  - building new lines
  - building switches
  - building microgrid facilities
  - building microgrid generation capacity

$\Rightarrow$ binary decisions, mixed-integer programming.

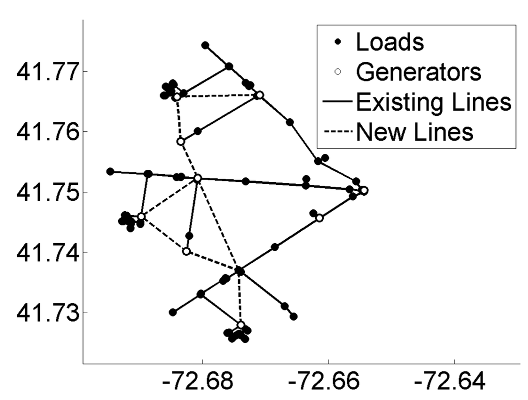
Nomenclature

Parameters

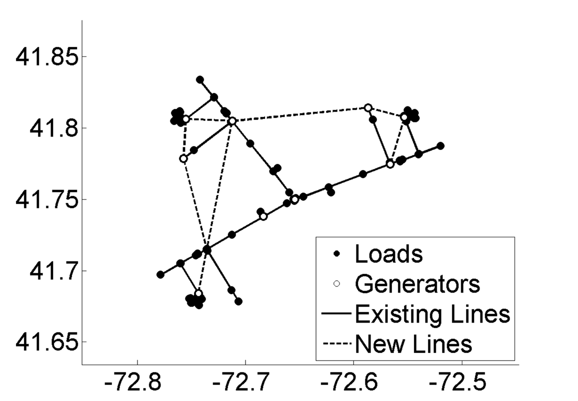
- $\mathcal{N}$ set of nodes (buses).
- $\mathcal{E}$ set of edges (lines and transformers).
- $\mathcal{S}$ set of disaster scenarios.
- $\mathcal{D}_s$ set of edges that are inoperable during $s \in S$.
- $\mathcal{D}'_s$ set of hardened edges that are inoperable during disaster $s \in S$.
- c_{ij} cost to build a line between bus i and j. 0 if line already exists.
- κ_{ij} cost to build a switch on a line between bus i and j.
- ψ_{ij} cost to harden a line between bus i and j.
- $\zeta_{i,k}$ cost of generation capacity on phase k at bus i.
- α_i cost to build a generation facility at node i.
- Q_{ijk} line capacity between bus i and bus j on phase k.
- $\mathcal{P}_{ij}$ set of phases for the line between bus i and bus j.
- $\mathcal{P}_i$ set of phases allowed to consume or inject at bus i.
- β_{ij} parameter for controlling maximum flow variation between the phases.
- $d_{i,k}$ demand for power at bus i for phase k.
- $G_{i,k}$ existing generation capacity on phase k at node i.
- $Z_{i,k}$ maximum amount of generation capacity on phase k that can be built at node i.
- $\mathcal{C}$ the set of sets of nodes that includes a cycle.
- λ fraction of critical load that must be served.
- γ fraction of all load that must be served.
- $\mathcal{L}$ set of buses whose load is critical.

Variables

- x_{ij} determines if line i,j is built.
- τ_{ij} determines if line i,j has a switch.
- t_{ij} determines if line i,j is hardened.
- $z_{i,k}$ determines the capacity for generation on phase k at node i.
- u_i determines the generation capacity built at node i.
- x^s_{ij} determines if line i,j is used during disaster s.
- τ^s_{ij} determines if switch i,j is used during disaster s.
- t^s_{ij} determines if line i,j is hardened during disaster s.
- $z^s_{i,k}$ determines the capacity for generation on phase k at bus i during disaster s.
- u^s_i indicates if the generation capacity is used at node i during disaster s.
- $g^s_{i,k}$ generation produced for bus i on phase k during disaster s.
- $l^s_{i,k}$ load delivered at bus i on phase k during disaster s.
- $y^s_{i_j}$ determines if the jth load at bus i is served or not during disaster s.
- $f^s_{ij,k}$ flow between bus i and bus j on phase k during disaster s.
- $\bar{x}^s_{ij}$ determines if at least one edge between i and j is used during disaster s.
- $\bar{\tau}^s_{ij}$ determines if at least one switch between i and j is used during disaster s.
- $x^s_{ij,0}$ determines if there exists flow on line i,j from j to i, during disaster s.
- $x^s_{ij,1}$ determines if there exists flow on line i,j from i to j, during disaster s.



(a) Urban



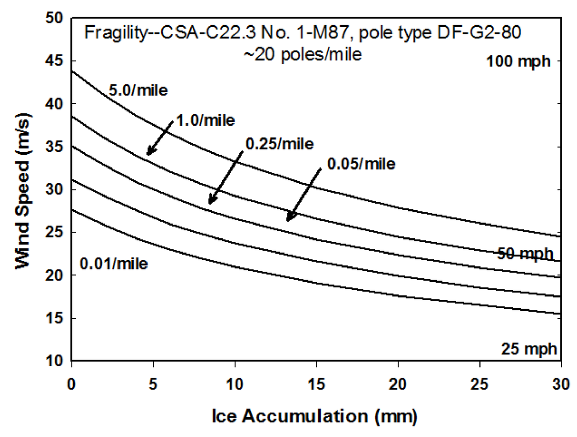
(b) Rural

Figure : Each problem contains three copies of the IEEE 34 system to mimic situations where there are three normally independent distribution circuits that could support each other during extreme events. These problems include 100 scenarios, 109 nodes, 118 possible generators, 204 loads, and 148 edges, resulting in problems with $> 90k$ binary variables. The cost of single and three phase underground lines is between $\$40k$ and $\$1500k$ per mile and we adopt the cost of $\$100k$ per mile and $\$500k$ per mile, respectively. The cost of single and three phase switches is estimated to be $\$10k$ and $\$15k$, respectively. Finally, the installed cost of natural gas-fired CHP in a microgrid is estimated to be $\$1500k$ per MW.

Scenario Definition

We assume each scenario can be associated to a subset of the lines of the power distribution system that are inoperable:

(c) Damaged line



(d) Fragility

Resilient Distribution Grid Design

$$
\begin{aligned}
\min \quad & \sum_{ij \in \mathcal{E}} c_{ij} x_{ij} + \sum_{ij \in \mathcal{E}} \kappa_{ij} \tau_{ij} + \sum_{ij \in \mathcal{E}} \psi_{ij} t_{ij} \\
& + \sum_{i \in \mathcal{N}} \alpha_i u_i + \sum_{i \in \mathcal{N}, k \in \mathcal{P}_i} \zeta_{i,k} z_{i,k} \\
\text{s.t.} \quad & x^s_{ij} \leq x_{ij} && \forall ij \in \mathcal{E}, s \in \mathcal{S} \\
& \tau^s_{ij} \leq \tau_{ij} && \forall ij \in \mathcal{E}, s \in \mathcal{S} \\
& t^s_{ij} \leq t_{ij} && \forall ij \in \mathcal{E}, s \in \mathcal{S} \\
& z^s_{i,k} \leq z_{i,k} && \forall i \in \mathcal{N}, k \in \mathcal{P}_i, s \in \mathcal{S} \\
& u^s_i \leq u_i && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& z_{i,k} \leq M_{i,k} u_i && \forall i \in \mathcal{N}, k \in \mathcal{P}_i \\
& (x^s, \tau^s, t^s, z^s, u^s) \in \mathcal{Q}(s) && \forall s \in \mathcal{S} \\
& x, \tau, t, u \in \{0, 1\}
\end{aligned}
$$

Minimize budget

1st stage: construction

2nd stage: assets in use flow delivery

Set of Feasible Distribution Networks

$$
\begin{aligned}
\mathcal{Q}(s) = \{ & x^s, \tau^s, t^s, z^s, u^s : \\
& -x^s_{ij,0} Q_{ijk} \leq f^s_{ij,k} \leq x^s_{ij,1} Q_{ijk} && \forall ij \in \mathcal{E}, k \in \mathcal{P}_{ij} \\
& x^s_{ij,0} + x^s_{ij,1} \leq x^s_{ij} && \forall ij \in \mathcal{E} \\
& (\tau^s_{ij} - 1) Q_{ijk} \leq f^s_{ij,k} \leq (1 - \tau^s_{ij}) Q_{ijk} && \forall ij \in \mathcal{E}, k \in \mathcal{P}_{ij} \\
& \frac{\sum_{k \in \mathcal{P}_{ij}} f_{ij,k}}{\frac{|\mathcal{P}_{ij}|}{(1-\beta_{ij})}} \leq f^s_{ij,k'} \leq \frac{\sum_{k \in \mathcal{P}_{ij}} f_{ij,k}}{\frac{|\mathcal{P}_{ij}|}{(1+\beta_{ij})}} && \forall ij \in \mathcal{E}, k' \in \mathcal{P}_{ij} \\
& x^s_{ij} = t^s_{ij} \leq \begin{cases} 0 & \text{if } ij \in \mathcal{D}'_s \\ 1 & \text{else} \end{cases} && \forall ij \in \mathcal{D}_s \\
& l^s_{i,k} = \sum_{j=0}^{n_i} y^s_{i_j} d_{i_j,k} && \forall i \in \mathcal{N}, k \in \mathcal{P}_i \\
& 0 \leq g^s_{i,k} \leq z^s_{i,k} + G_{i,k} && \forall i \in \mathcal{N}, k \in \mathcal{P}_i \\
& g^s_{i,k} - l^s_{i,k} - \sum_{j \in \mathcal{N}} f^s_{ij,k} = 0 && \forall i \in \mathcal{N}, k \in \mathcal{P}_i \\
& 0 \leq z^s_{i,k} \leq Z_{i,k} u_i && \forall i \in \mathcal{N}, k \in \mathcal{P}_i \\
& \sum_{ij \in \mathcal{E}(C)} (x^s_{ij} - \tau^s_{ij}) \leq |V| - 1 && \forall C \in \mathcal{C} \\
& \tau^s_{ij} \leq x^s_{ij} && \forall ij \in \mathcal{E} \\
& \sum_{i \in \mathcal{L}, k \in \mathcal{P}_i} l^s_{i,k} \geq \lambda \sum_{i \in \mathcal{L}, k \in \mathcal{P}_i} d_{i,k} \\
& \sum_{i \in \mathcal{N} \setminus \mathcal{L}, k \in \mathcal{P}_i} l^s_{i,k} \geq \gamma \sum_{i \in \mathcal{N} \setminus \mathcal{L}, k \in \mathcal{P}_i} d_{i,k} \\
& x^s, y^s, \tau^s, u^s, t^s \in \{0, 1\} \}
\end{aligned}
$$

3-phase real flow

Modeled as multicommodity flow with phase variation

Hardened lines can still be damaged

Distribution network $\Rightarrow$ tree

Minimum service requirement as resilience criteria

Urban, Hardened lines are not damageable (a)

	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	19984.7	322.9	1044.5	465.8	322.9	289.9	353.7
25%	166352	635.4	1643.5	8028.3	635.4	811.4	635.4
50%	TO	X	2021.2	2840.7	647.7	791.3	647.7
75%	TO	X	1874.2	991.1	652.1	692.5	652.1
100%	TO	X	1934.4	712.7	654.1	662.5	654.1

Rural, Hardened lines are not damageable (b)

	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	33083.5	2337.0	3274.8	1837.9	2337.0	503.3	2337.0
25%	32170.8	2390.3	3427.6	571.0	2390.3	457.8	2390.3
50%	20840.3	2397.6	3449.9	471.2	2397.6	421.2	2397.6
75%	15556.1	2400.4	3452.7	337.5	2400.4	299.8	2400.4
100%	17225.9	2400.6	2780.6	385.8	2400.6	346.9	2400.6

Urban, Hardened lines are damaged at a $\frac{1}{100}$ rate (c)

	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	159166	445.8	1061.7	2232.9	445.8	2721.3	476.5
25%	TO	X	1441.9	14299.2	662.9	2994.7	701.5
50%	TO	X	1571.2	2848.7	646.0	1917.7	760.2
75%	TO	X	1787.3	16040.6	687.6	1481.4	687.6
100%	TO	X	2744.8	24270.3	1320.5	2157.5	1330.5

Rural, Hardened lines are damaged at a $\frac{1}{100}$ rate (d)

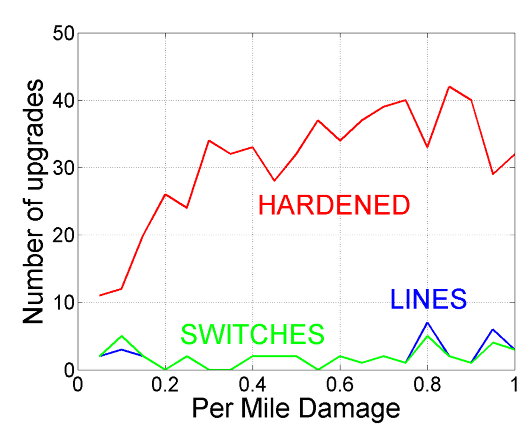
	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	77947.9	2363.0	3375.4	759.0	2363.0	576.9	2363.0
25%	TO	X	8238.6	TO	X	919.4	6744.3
50%	TO	X	12336.0	TO	9288.9	4361.8	7121.0
75%	TO	X	23099.5	TO	X	23142.6	11500.0
100%	TO	X	16600.7	TO	X	5879.5	9797.3

Urban, Hardened lines are damaged at a $\frac{1}{10}$ rate (e)

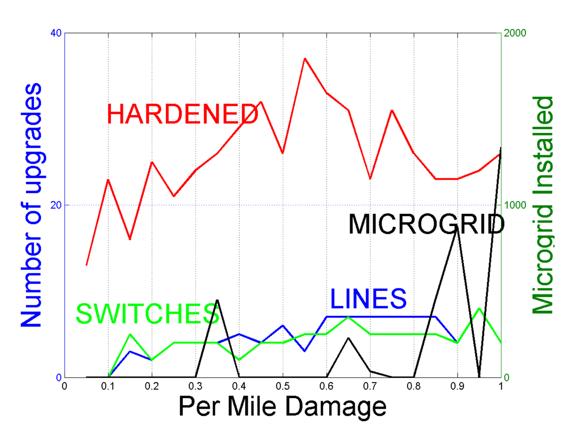
	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	TO	X	859.1	5265.1	460.8	2505.7	594.1
25%	TO	X	1742.2	12530.3	961.2	2843.2	961.2
50%	TO	X	3133.8	34822.7	1417.2	3363.5	1555.2
75%	TO	X	3472.0	TO	X	7486.5	1894.2
100%	TO	X	10479.1	TO	X	32289.8	7959.4

Rural, Hardened lines are damaged at a $\frac{1}{10}$ rate (f)

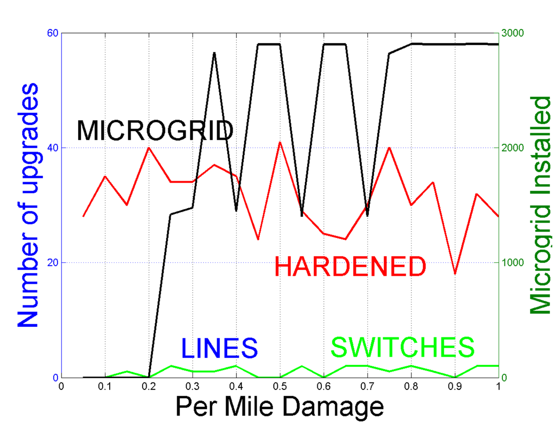
	CPLEX		Greedy	SBD		SBVNDS	
	CPU	OBJ	OBJ	CPU	OBJ	CPU	OBJ
10%	TO	X	7503.3	141718.0	4325.9	7756.8	4424.8
25%	TO	X	18021.3	TO	X	21993.5	7371.9
50%	TO	X	28865.0	TO	12017.7	74729.0	12031.2
75%	TO	X	31887.0	TO	13522.2	107165.0	13500.8
100%	TO	X	32901.9	TO	16794.4	114354.0	16778.2



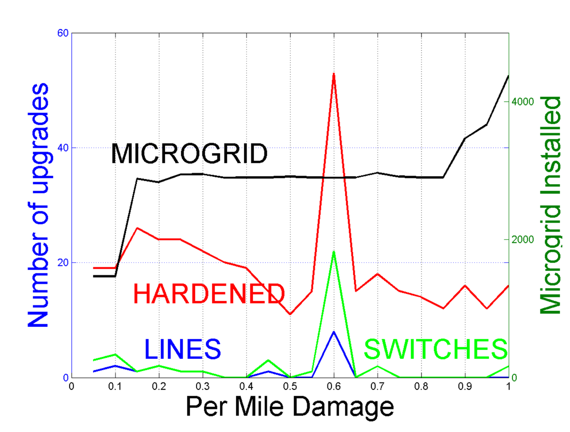
(a) Urban - 1/100 Rate



(b) Urban - 1/10 Rate



(e) Rural - 1/100 Rate



(f) Rural - 1/10 Rate